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# 1.0 Project Description:

This model is a swap pricing engine. It is used to compute Swap rate for a plain vanilla interest rate swap, which is the fixed interest rate on the fixed leg of a swap. Additionally, the output includes a swap pricing engine that helps to calculate the value of the swap after ‘t’ from the swap initiation. The model takes SOFR rate curve as inputs and produces Swap rate through boot strapping.

Background on the rates:

* **SOFR Rate**: The SOFR rate helps to estimate the cost of collateralized borrowing in the U.S Repo market. It is an overnight borrowing rate. Let us say the 6M SOFR rate is 4% and a bank wants to borrow $100 million, then it costs $2 million ($4million/2) to borrow $100 million through the repo. The bank has pay $2 million for every six to do this collateralized borrowing.
* **Interpreting 3M/6M/12M SOFR rate**: 3M sofr is effectively calculated from daily SOFR rate. It is equivalent annual rate when you do SOFR borrowing overnight and roll over for 3M.
* : This is forecasted 12m SOFR rate at time t. For example, represents the forecasted 12m sofr rate at time t=1year.
* **Swap Rate:** The swap rate helps to identify the cost of the hedging. It is the interest rate on the fixed payment leg of an interest rate swap. Let us say, 1 year swap rate is 4.5% and a bank wants to hedge $100 million variable loan exposure. Then the bank would pay $4.5 million to hedge this variable rate loan. The bank pays $2.25 million (4.5%/2) for every six months and receives 3M SOFR rate per quarter.

# 2.0 Methodology

## 2.1 Swap pricing

At initiation, the value of swap is zero.

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As per equation (1)

Where

represents the discount factor for period.

represents the fixed rate (swap rate)

represents the present value of the final notional.

Solving this equation, 1 year swap rate is computed as

---(2)

## 2.2 Swap valuation

= sum of discounted rates (swap discount rate at time t)

## 2.3 Swaption pricing

A swaption is simply having an option to enter into the swap when interest rates move in your favor. For example, let us say JPMC entered into an interest rate swap as a fixed rate payer at 4%. Then interest rates moved down by 3%. Then in this scenario, JPMC would lose 1% on the swaption because of a decrease in interest rates. However, the loss will be limited to premium with a swaption.

A swaption requires an interest rate model. Hull White model and HJM are considered to be good options for interest modeling. HJM is selected over Hull model because of long-term interest rate capability of HJM.

Per Equation 2, swap price is first determined

Then valuation of the option at each path is computed as follows:

Payer swaption:

Receiver swaption:

Take average of the option values across all simulated paths:

# 3.0 Model output

The model out has three components: (i) Swap pricing (Fixed rate payer) (ii) Swap valuation (iii) Swaption pricing (MonteCarlo simulation)

|  |  |  |
| --- | --- | --- |
| Component | Description | Remarks |
| Swap pricing | The fixed interest rate on for fixed rate payment leg |  |
| Swap valuation | Change in the swap price (output 1) with the passage of time |  |
| Swap pricing | Value of the swaption at a point in time | * HJM model [4] is used for simulating the interest rate paths   #number of simulations:100   * See output below |

   double volatility = 0.002;

    double delta\_t = 0.25;

    int steps = 4;

    int num\_of\_simulations = 100;

    unsigned seed = 95;

To reduce the complexity, the parameters for HJM model (interest rate curve) are directly taken as inputs. In reality, the parameters will be calibrated.

HJM model output (3m,6m,9m,12m)

A screen shot of a computer

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Illustrative example to understand the interest rate swaption price

* Discount factors:

D(3M)=0.987995,

D(6M)=0.975846

,D(9M)=0.963558,

D(12M)=0.951229D

* Δt=0.25
* Swap rate S=4.728915%
* Exercise price X=4.7%
* Notional N=1,000,000.

**Steps:**

1. Compute PV​:

PV=(0.987995+0.975846+0.963558+0.951229)⋅0.25 = 0.969157

1. Swap rate – exercise rate = 0.0289%
2. Payoff =

Compute the payoff: Payoff=0.00028915⋅0.969157⋅1,000,000=($) 280.15

# 4.0 Implementation

The model produces the swap rate in three steps. First, a sofr curve is produced taking the current sofr rate as the input. Second, sofr rate curve is converted into a discountrate curve. The discount rates are calculated through the function getDiscountFactors() defined within the class “swaprate calculator”. Third, the swap rate is computed as per the mathematical formula details in the equation 2. This code is available on Git for ready reference.

* Key methodology step implementation:

The following image shows the key implementation step in the swap engine. It can be observed from the image that the swap rate “swap\_rate\_quarterly” is derived from the discount factors.

Figure 0‑1 Implementation of discounting factors

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Source: C++ code [2]

* Swaprate calculator class

The following image shows the swap engine class implementation in c++. The function calculateDiscountFactors is responsible for calculating the discount factors given a rate curve.

Figure 0‑2 Implementation of swaprate calculator class

A computer screen shot of a program

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Source: C++ code [2]

**Swap valuation logic**

Loss=−0.6238%×Notional×PV (3.893).

1. Interest Rate Changes:
   * The fixed rate decrease (from 4.8395% to 4.2157%) means the obligation to pay the fixed leg is now higher than what the market would demand today for a similar swap.
   * This directly results in a valuation loss.
2. PV Sum (3.893):
   * This PV sum represents the discounted cash flows over the remaining one year of your swap. It reflects the time value of money for the remaining payments.
3. Valuation Formula:
   * The valuation loss reflects the economic impact of paying a now unfavorable fixed rate relative to market levels: Valuation Loss=−0.006238×Notional×3.893

**Example:**

If your notional is $1,000,000:

Loss=−0.006238×1,000,000×3.893=−$24,272.57

The P&L impact is −$24,272.57 because of interest rate changes

Figure 0‑3 Implementation of swap valuation logic

A screen shot of a computer code

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Figure 0‑4 Swaption price

A computer code on a black background

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A screen shot of a computer code

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Figure 0‑5Model output from Run

A screenshot of a computer program

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Figure 0‑6 Swap rate for each simulation

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# 5.0 Future work

* Calibrate the interest rate curve by fitting to the current g-sec curve. [Out of scope for this project]

# References

[1] Federal Reserve, "Historical Overnight SOFR Rate." Available: <https://fred.stlouisfed.org/series/SOFR>.

[2] M. Matala and Arya, "C++ code swap rate." Available: <https://github.com/manojmatala/swapvaluationengine>.

[3] US Treasury Yield curve

U.S. Department of the Treasury, "Daily Treasury Yield Curve Rates," Accessed: Nov. 10, 2024. [Online]. Available: <https://home.treasury.gov/resource-center/data-chart-center/interesrates/TextView?type=daily_treasury_yield_curve&field_tdr_date_value=2024>

[3] Git hub code availability

A screenshot of a computer

Description automatically generated

[4] D. Heath, R. Jarrow, and A. Morton, "Bond Pricing and the Term Structure of Interest Rates: A New Methodology," *Econometrica*, vol. 60, no. 1, pp. 77–105, Jan. 1992. doi: 10.2307/2951677